

Measurement of The Spring Elastic Constant “k”

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Abstract

This laboratory report was written to report on two experiments that were carried out to measure the elastic constant “ k ” of a spiral spring, the first experiment used Hooke’s Law to measure the constant k , while the second experiment used the oscillation period of the spring to measure the constant k . The first experiment gave $k = 5.88 \frac{N}{m}$, while the second experiment gave $k = 5.87 \frac{N}{m}$, and the difference is reasonable due to the slight inaccuracy of the measurement tools.

Introduction

This Experiment is divided into two parts: in the first part, the aim is to measure the elastic constant of a spring using Hooke’s Law, while the second part aims to measure the same spring’s elastic constant using the oscillations of the spring.

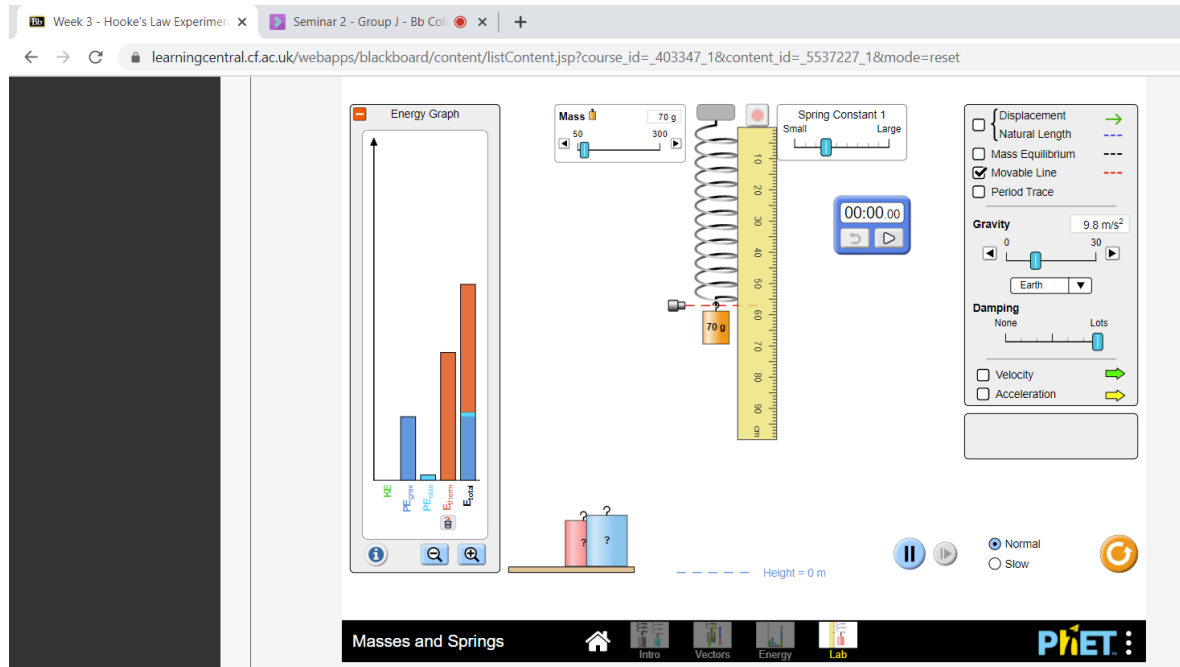
First Experiment’s Theory: Hooke’s Law states that if a spring was extended or compressed by a force of magnitude F , the extension or compression of the spring is related to F by the relationship $F = -kx$ where x is the compression or extension, and k is the elastic constant of the spring (Halliday, Resnick and Walker, 2014), this equation is a linear equation, that we can use to find k .

Second Experiment’s Theory: If a mass was attached to a vertical spring, and that mass was made to fall due to gravity, then the restoring force given by Hooke’s law makes the mass oscillate with a period $T = 2\pi\sqrt{m/k}$ where m is the attached mass, T is the period “the time the oscillating mass take to complete a full oscillation”, and k is the elastic constant of the spring (Halliday, Resnick and Walker, 2014), squaring this equation and rearranging would result in a linear relationship between T and M , and we can use this relationship to find k .

It is predicted that the two experiments show the same results, as the spring was not changed “ k is a property of the spring itself”, and as the theories for them are connected “the period equation can be derived from Hooke’s Law, but this requires solving differential equations”

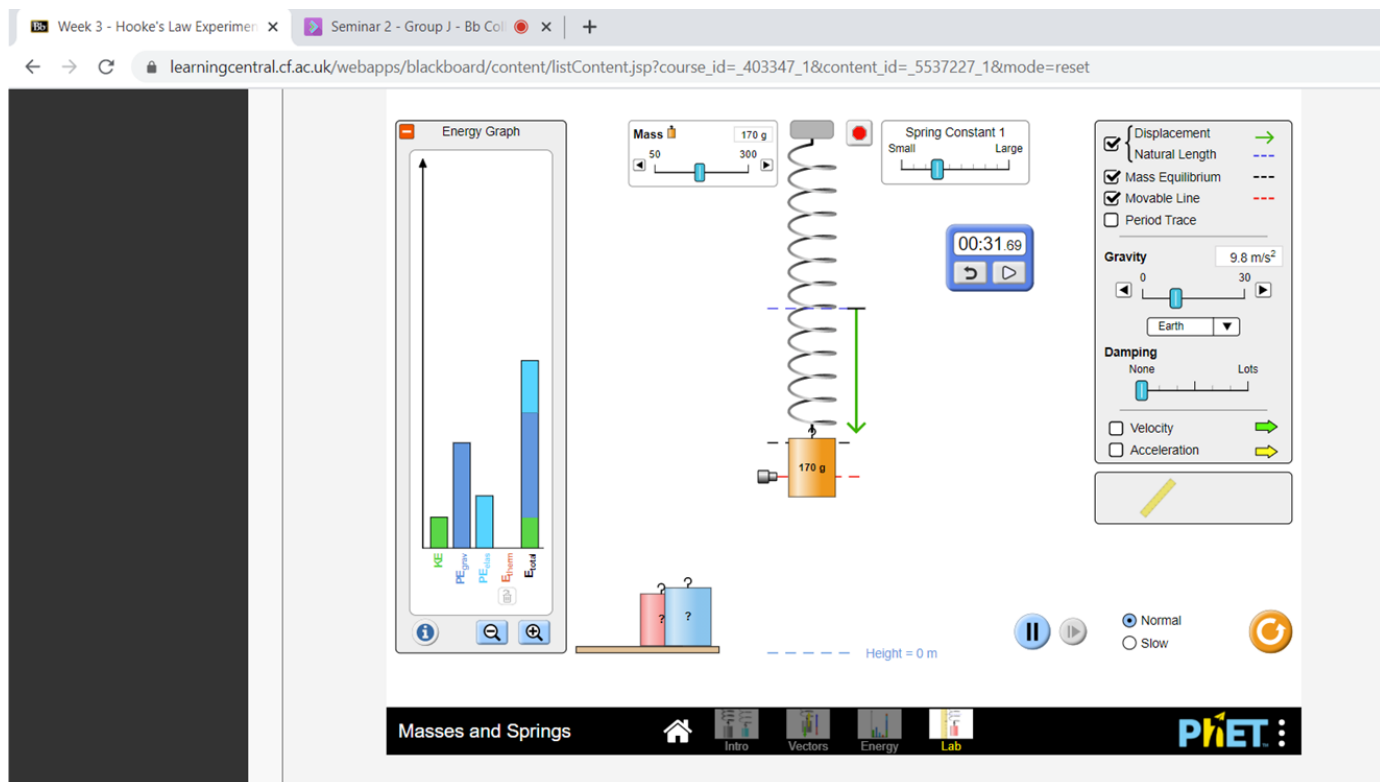
Method

The first experiment was done virtually on the simulation at Learning Central Platform, and in this virtual experiment, the following equipment was provided: A Light Spiral Spring, a Ruler, a Pointer, and a Set of weights. And these equipment were assembled as in the below snapshot of the experiment.



the length of the spring without any weights was measured and recorded, and then the weights were attached one by one, with a slight change in the mass each time, and the length of the spring was measured each time, and the mass of the weight, along with the length of the loaded spring was recorded in the logbook, the ruler reading was converted to meters and recorded, also the weight “Force that the gravity exerts on the mass” was calculated and recorded.

For the second experiment, it was the same as the first one “in terms of place and equipment”, apart from using a timer to measure the period, and the damping was lowered to allow the oscillations to happen (see the Experiment screenshot below for setup).



The spring was loaded with weights from 100 grams to 170 grams, the time for 30 periods was measured and the time for each weight was recorded in the logbook, along with the load “the attached mass” of the spring. The Period T of the oscillations was obtained by dividing the measured time by 30, then the period T was recorded along with T^2 , which is the Square of the Period T .

Analysis and Results

The Results for the first experiment was as in the below table:

Table 1: Results of the First Experiment

Mass (g)	Ruler Reading (cm)	Ruler Reading(m)	Weight (N)
0	46	0.46	0
50	54	0.54	0.4905
70	57	0.57	0.6867
90	60	0.60	0.8829
110	64	0.64	1.0791
130	67	0.67	1.2753
150	70	0.70	1.4715
170	74	0.74	1.6677
190	76	0.76	1.8639
210	81	0.81	2.0601

A plot of the Weight on the x-axis against the Ruler Reading (in meters) on the y-axis was made using GeoGebra, then a best-fit line for the Points was drawn, and the result is illustrated in the graph below.

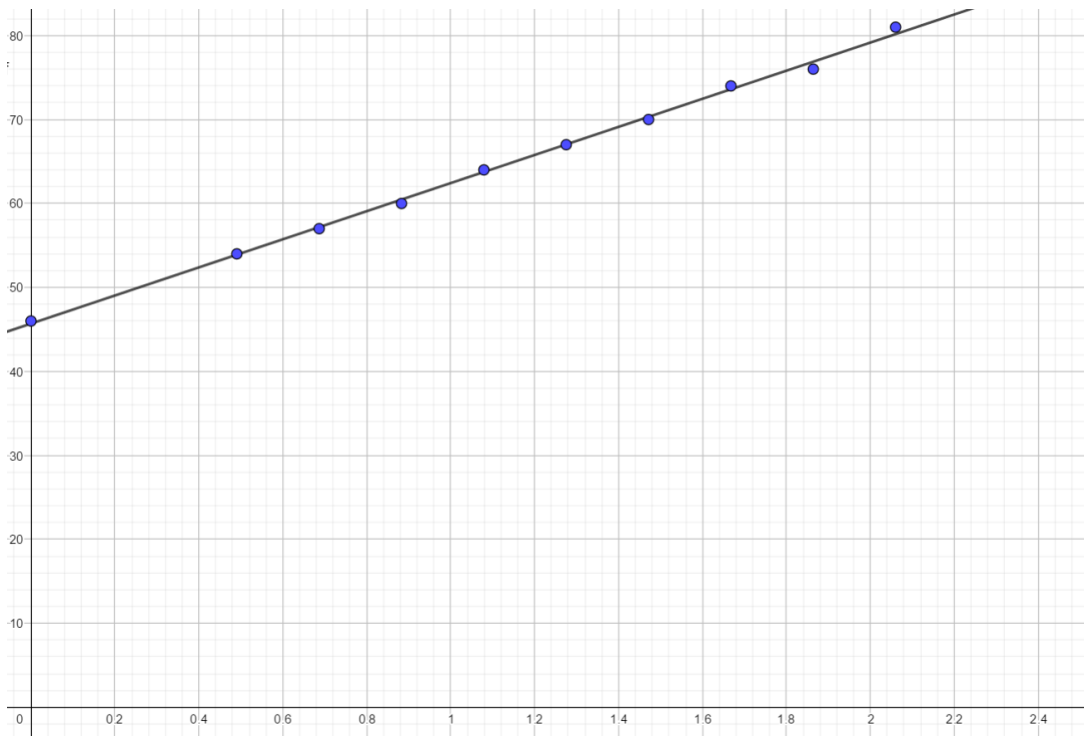


Figure 1: Plot of the Results (first experiment)

The Analysis of the result is as follows:

The extension of the spring is $x = l - l_0$ where l is the length of the extended spring, and l_0 is the natural length of the spring, so Hooke's Law will become $F = k(l - l_0) = kl - kl_0$.

We know that the only Force Acting on the spring, other than the Restoration Force given by Hooke's Law, is the Weight of the attached mass, so $F = W$ thus $W = kl - kl_0$.

Rearranging the last equation gives $W + kl_0 = kl$, then after dividing by k , we get $l = \frac{W}{k} + l_0$, so our line graph is a line with slope $\frac{1}{k}$ and y-intercept l_0 .

GeoGebra Gives the line's equation as $l = 0.17W + 0.46$, so the slope is $0.17m/N$, and the Elastic Constant is $k = \frac{1}{\text{slope}} = \frac{1}{0.17} = 5.88 \text{ N/m (3 s.f.)}$.

For the second experiment, the results are as in the table below.

Table 2: Results of the second Experiment

Load (Kg)	Time for 30 Oscillations (s)	Period T (s)	T ² (s ²)
0.1	24.46	0.815333333	0.664768444
0.11	25.68	0.856	0.732736
0.12	26.72	0.890666667	0.793287111
0.13	27.77	0.925666667	0.856858778
0.14	28.89	0.963	0.927369
0.15	30.51	1.017	1.034289
0.16	31.06	1.035333333	1.071915111
0.17	31.69	1.056333333	1.115840111

Again, a plot of the load m in kilograms on the x-axis and T^2 in s^2 on the y-axis was made using GeoGebra, then the best-fit line was created, and the result is illustrated in the below picture

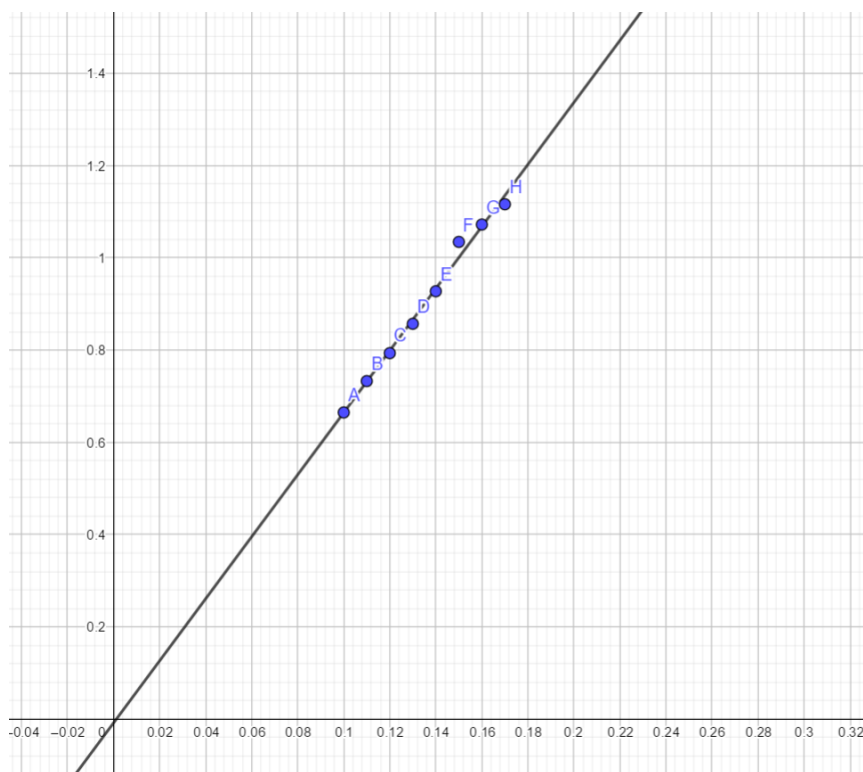


Figure 2: Results of the Second Experiment

The analysis is as follows:

The Period of the oscillations is $T = 2\pi\sqrt{m/k}$, squaring both sides, we get $T^2 = 4\pi^2 m/k$, so the graph of T^2 against m is a straight line with slope $\frac{4\pi^2}{k}$.

GeoGebra gives the line equation as $T^2 = 6.72m$, so $slope = \frac{4\pi^2}{k} = 6.72$, thus $k = \frac{4\pi^2}{slope} = \frac{4\pi^2}{6.72} = 5.87 \text{ N/m}$ (3 s. f.).

Discussion

As the spring was not changed between the two experiments, the obtained values of k should be the same in the two experiments, but a slight difference of 0.01 N/m occurred because of the timing in the second experiment, and the low accuracy of the ruler in the first experiment.

Also, the data set for the Second experiment was smaller than the first, this is also a reason that makes for the difference between the two experiments.

Through that reasoning, there was no unexpected results, and the theoretically predicted results match with the experimental results.

Conclusion

To conclude, the two experiments was carried to measure the Elastic constant of the spring in two different methods, then answering the question if they are consistent or not, the final findings of the experiment were that the first method gave a result of 5.88 N/m for the elastic constant and the second method shows that the elastic constant is 5.87 N/m for the same spring. Under the limitation that both the ruler and the stopwatch are a little inaccurate, these results are consistent, these two experiments, built an understanding of how Hooke's law and the simple harmonic oscillations in the spring are related.

References

Halliday, D., Resnick, R. and Walker, J. (2014) *Principles of Physics*, Singapore, Wiley.